Year 12 Mathematics Specialist 2019

Test Number 2: Functions and Graph Sketching

Resource Free

ALL SAINTS'
COLLEGE

Name: **SOLUTIONS**   Teacher: Mrs Da Cruz

Marks: **44**

Time Allowed: **45 minutes**

Instructions: You **ARE NOT** permitted any notes or calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

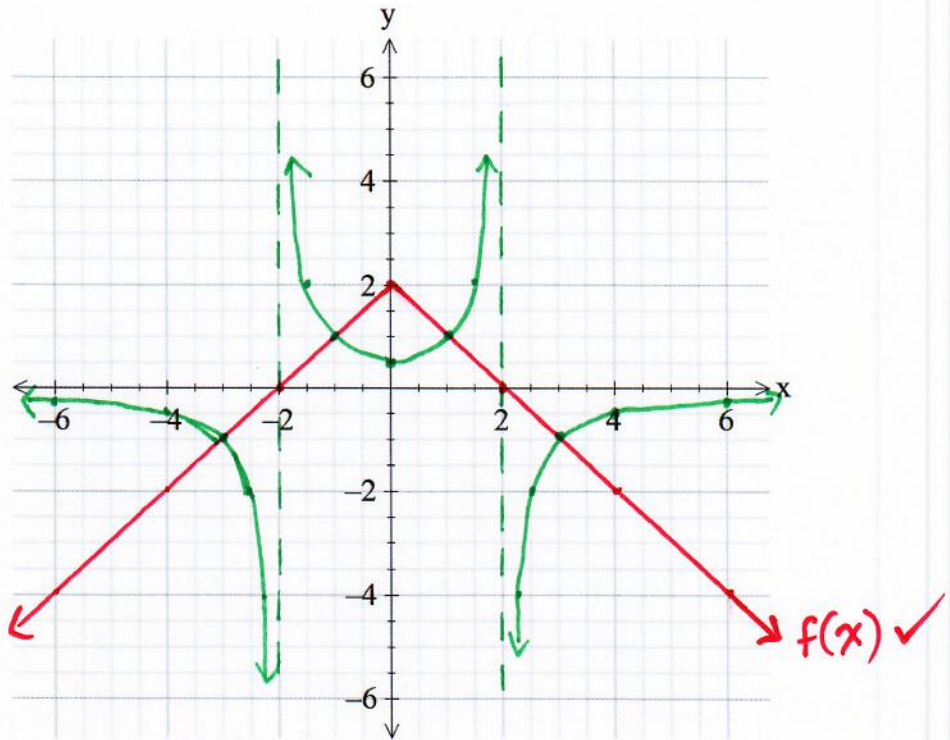**Question 1**   **[3, 3, 1, 2 = 9 marks]**

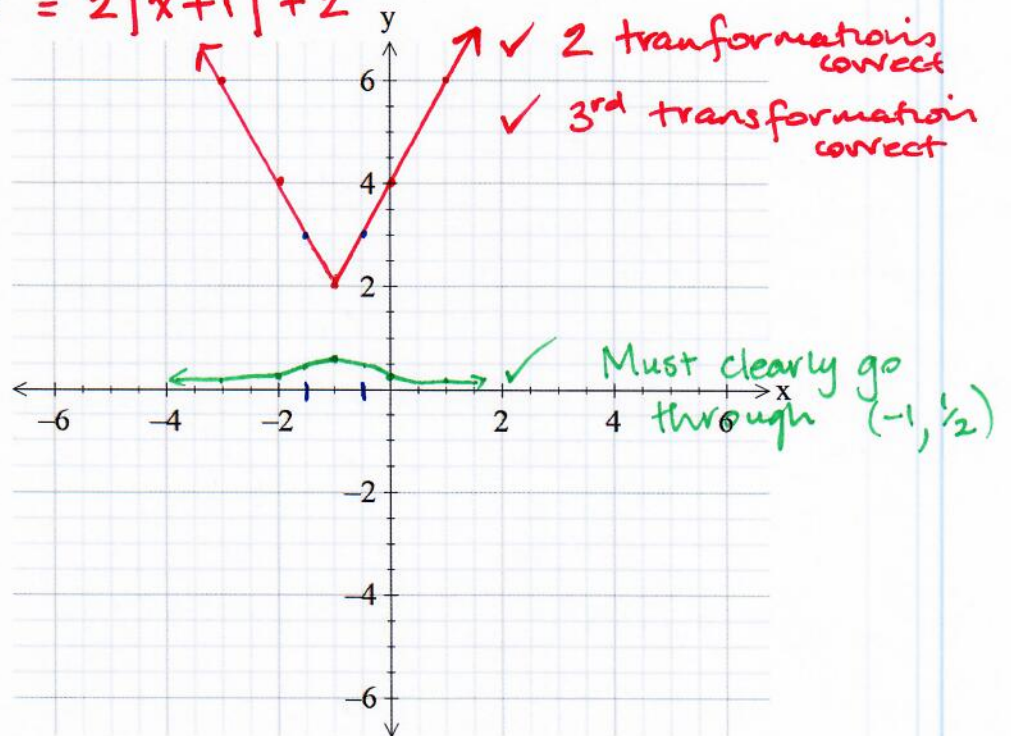Sketch the graphs of $f(x)$ and $\frac{1}{f(x)}$ on the same cartesian plane for each of the following:

a) $f(x) = -|x| + 2$

✓ v. asymptotes & $y=1$ points

✓ $2 \leftrightarrow \frac{1}{2}$ & accurate curves

$f(x)$ ✓

b) $f(x) = |2x + 2| + 2$ $= 2|x+1| + 2$

✓ 2 transformations correct

✓ 3rd transformation correct

✓ Must clearly go through $(-1, \frac{1}{2})$

c) Use your graphs in a) and b) to help you solve the following:

i) $0 = -|x| + 2$

        $x = \pm 2$ ✓

ii) $1 \leq |2x + 2|$

    $\therefore 3 \leq |2x+2| + 2$ $\left\{ x \leq -\frac{3}{2} \cup x \geq -\frac{1}{2} \right\}$

Question 2 [3, 2, 3 = 8 marks]

Given that $f(x) = \frac{2x-1}{3-x}$: $= \frac{-2x+1}{x-3}$

a) By rearranging $f(x)$ into the form $a + \frac{b}{x-3}$, find the asymptotes of $f(x)$.

$$3 \,|\, \begin{array}{cc} -2 & 1 \\ \downarrow & -6 \\ \hline -2 & -5 \end{array}$$

$$f(x) = -2 - \frac{5}{x-3} \checkmark$$
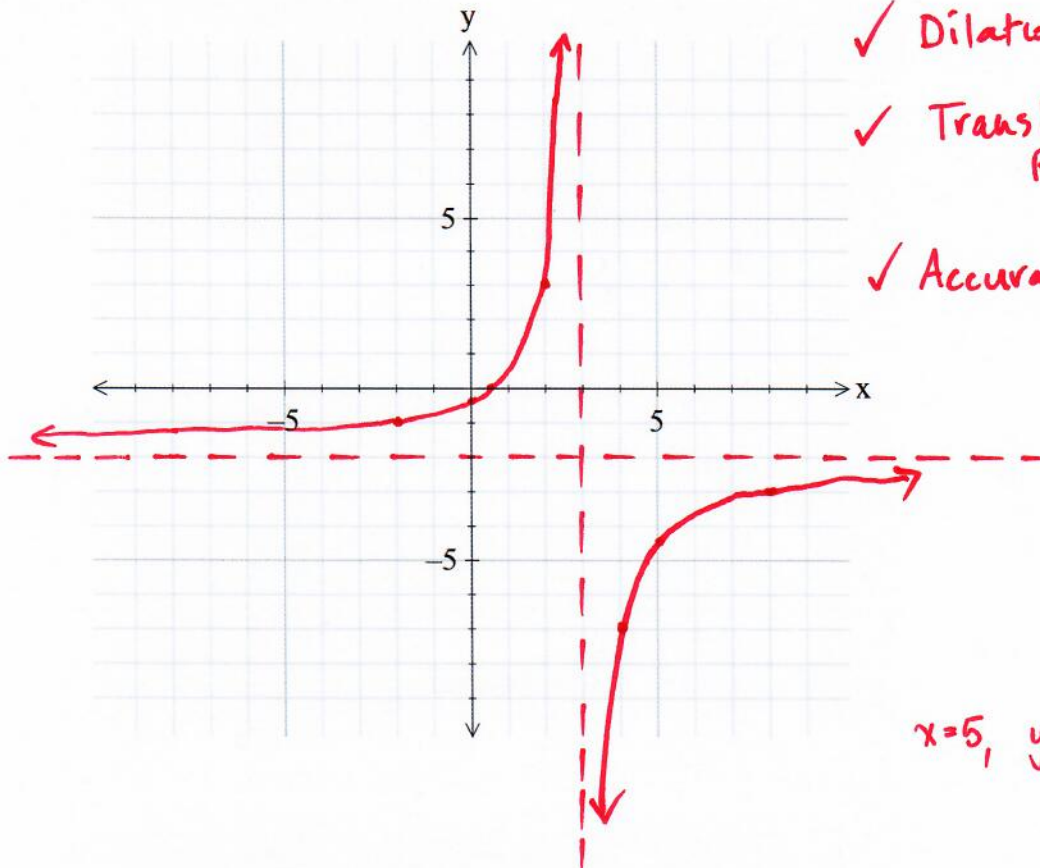
$\therefore$ Asymptotes at $x = 3$ ✓ and $y = -2$ ✓

b) Find the axes intercepts of $f(x)$.

$x = 0 \qquad y = -\frac{1}{3}$

$y = 0 \qquad 2x - 1 = 0$

$\qquad\qquad x = \frac{1}{2}$

Intersepts

$(0, -\frac{1}{3})$ ✓

$(\frac{1}{2}, 0)$ ✓

c) Sketch the graph of $f(x)$.

✓ Dilation $\times 5$

✓ Translations. & Reflection

✓ Accurate curves

$x = 5, \; y = \frac{9}{-2}$

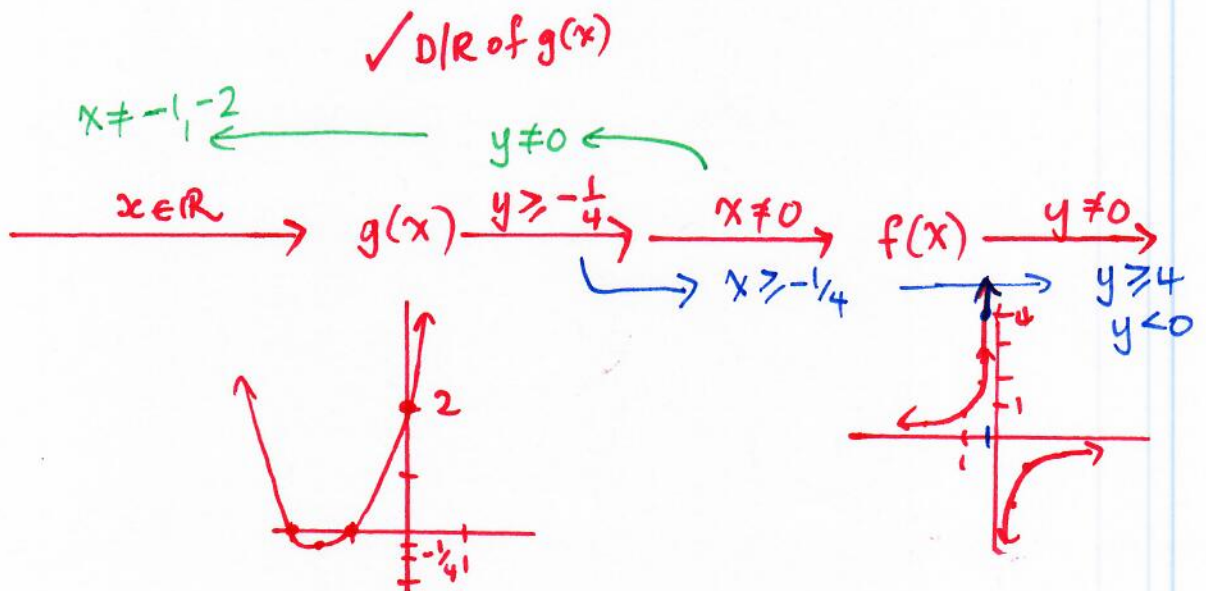**Question 3**   **[1, 4, 1, 2, 3, 1 = 12 marks]**

Given that $f(x) = -\frac{1}{x}$ and $g(x) = x^2 + 3x + 2$.

a) Find $fog(x)$.

$$- \frac{1}{x^2 + 3x + 2}$$

b) State the domain and range of $fog(x)$.

$\checkmark$ D/R of $g(x)$

$x \neq -1, -2$ $y \neq 0$

$x \in \mathbb{R} \longrightarrow g(x) \xrightarrow{y \geqslant -\frac{1}{4}} x \neq 0 \longrightarrow f(x) \xrightarrow{y \neq 0}$

$\longrightarrow x \geqslant -\frac{1}{4}$ $y \geqslant 4$
                                                        $y < 0$

$g(x) = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2$

$g(x) = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

Also $g(x) = (x+2)(x+1)$

or $x = -\frac{3}{2}$ T.P.
      $y = -\frac{1}{4}$

Domain : $\left\{ x \in \mathbb{R} : x \neq -1, x \neq -2 \right\}$

Range : $\left\{ y \in \mathbb{R} : y < 0 \cup y \geqslant 4 \right\}$

$\checkmark\checkmark\checkmark$

-1 per error

c) Are either of $f(x)$ or $g(x)$ one-to-one functions? If so, state which ones.

$$\text{Yes, } f(x) \text{ is } 1\text{-}1. \quad \checkmark$$

d) Find $f^{-1}(x)$ stating any restriction on the domain of $f(x)$ if required.

$$\therefore f^{-1}(x) = -\frac{1}{x} \quad \checkmark$$

$$y = -\frac{1}{x} \quad \Rightarrow \quad x = -\frac{1}{y}$$

$$\therefore y = -\frac{1}{x}$$

Domain of $f(x)$ is $\{x \in \mathbb{R} : x \neq 0\}$

No further restriction required $\checkmark$ (Doesn't have to be stated.)
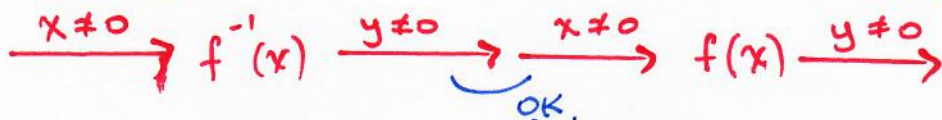
*Note question should have asked for explanation.

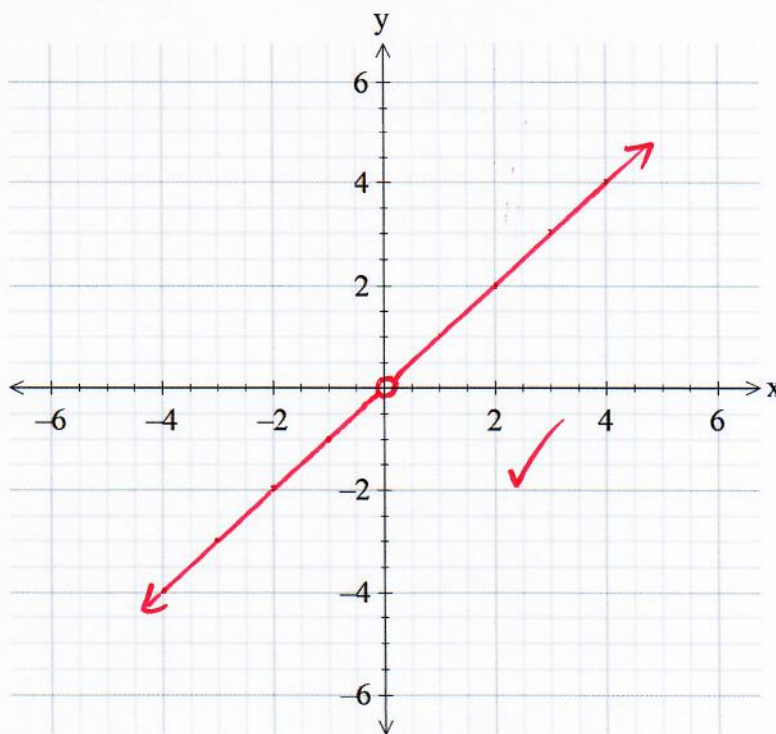e) State the domain and range of $f \circ f^{-1}(x)$. Find $f \circ f^{-1}(x)$.

$$f \circ f^{-1}(x) = -\frac{1}{-\frac{1}{x}} = x \quad \checkmark$$

DOMAIN $\checkmark$

$$\xrightarrow{x \neq 0} f^{-1}(x) \xrightarrow{y \neq 0} \xrightarrow{x \neq 0} f(x) \xrightarrow{y \neq 0}$$

O.K.

RANGE $\checkmark$

f) Sketch $f \circ f^{-1}(x)$.

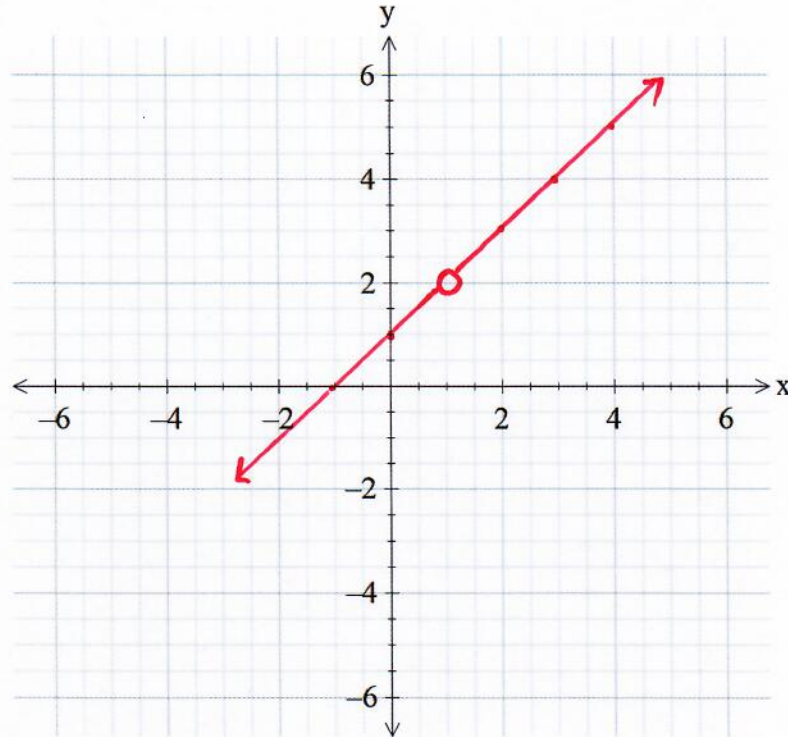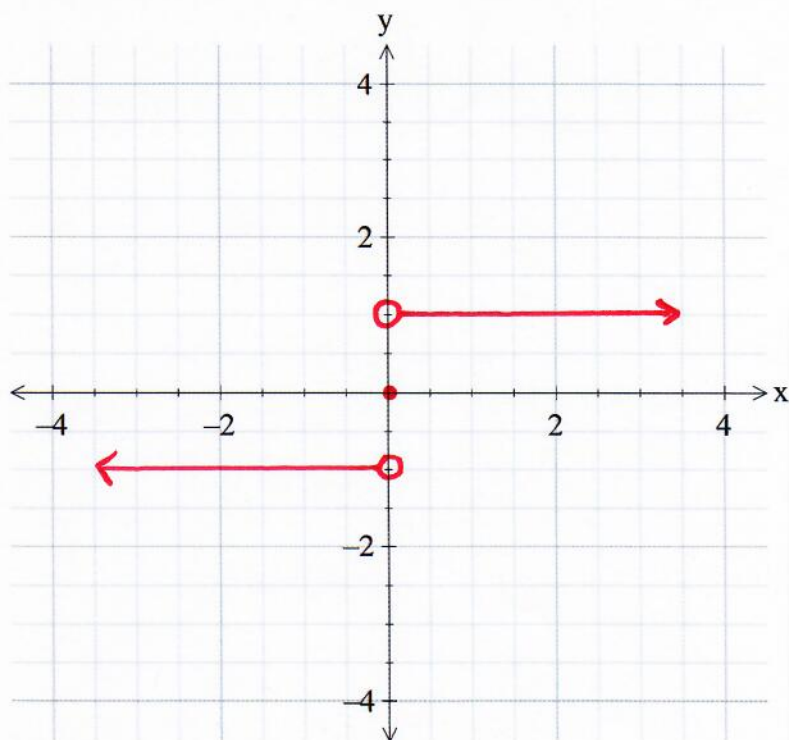**Question 4**   **[2, 2 = 4 marks]**

Sketch the following functions

a) $f(x) = \frac{x^2-1}{x-1}$ $=$ $\dfrac{(x-1)(x+1)}{x-1}$ $=$ $x+1$ ✓ ; $x \neq 1$ ✓

b)   The piecewise function called the Sign Function: $sgn(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$
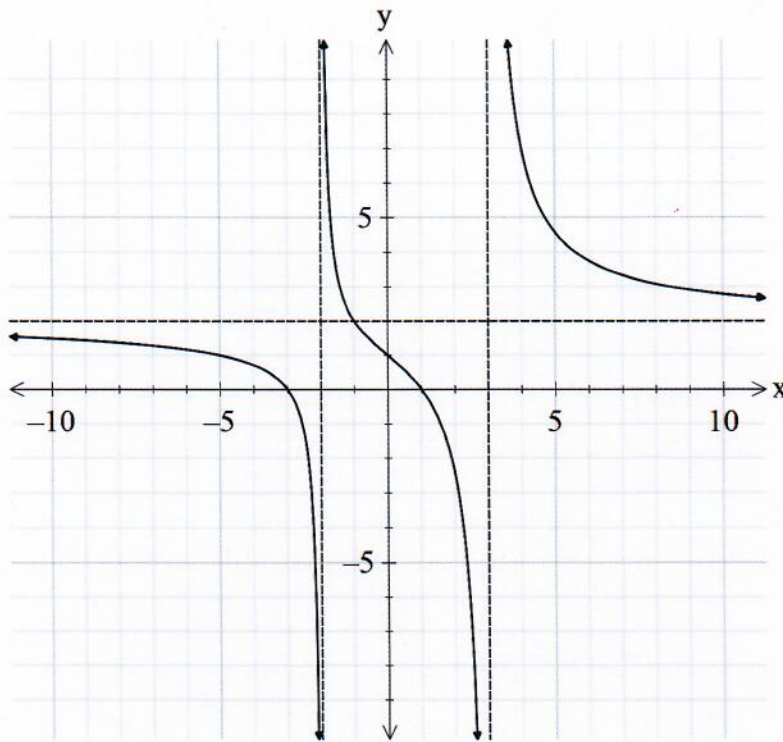

✓ $x=0$ ⦂

✓ 2 horiz. lines

Question 5 [4 marks]

The graph of $f(x) = \dfrac{k(x-a)(x-b)}{(x-c)(x-d)}$ is shown below.

$y = 0$ when $x = a$
$\qquad\qquad\qquad\quad x = b$

$\therefore a, b = -3, 1$ ✓

undef. when $x = c$
$\qquad\qquad\qquad x = d$

$\therefore c, d = -2, 3$ ✓

Determine the value of the constants a, b, c, d and k.

a	b	c	d	k
-3	1	-2	3	2

↖ or ↗   ↖ or ↗   ✓

Explain your choice for the value of k.

y-int:

$1 = \dfrac{k(0--3)(0-1)}{(0--2)(0-3)}$ ✓

$1 = \dfrac{k(-3)}{-6}$

$\therefore k = 2$

(or could use $(-5, 1)$ or $(4, 7)$)

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or as $x \to \pm\infty$, $y \to \dfrac{kx^2}{x^2} = k$ ✓
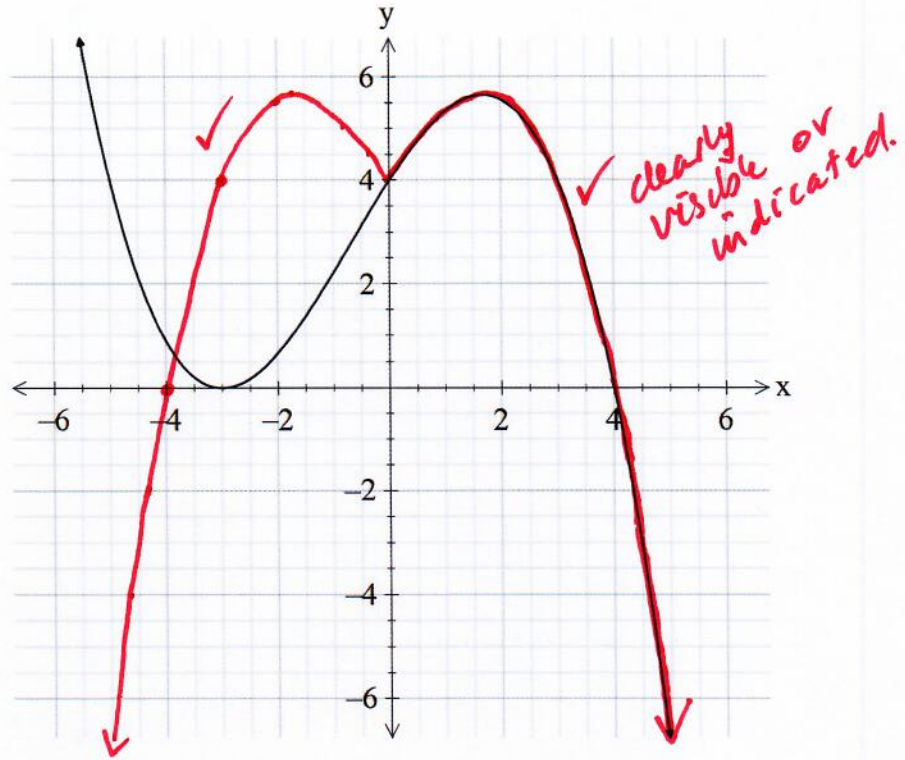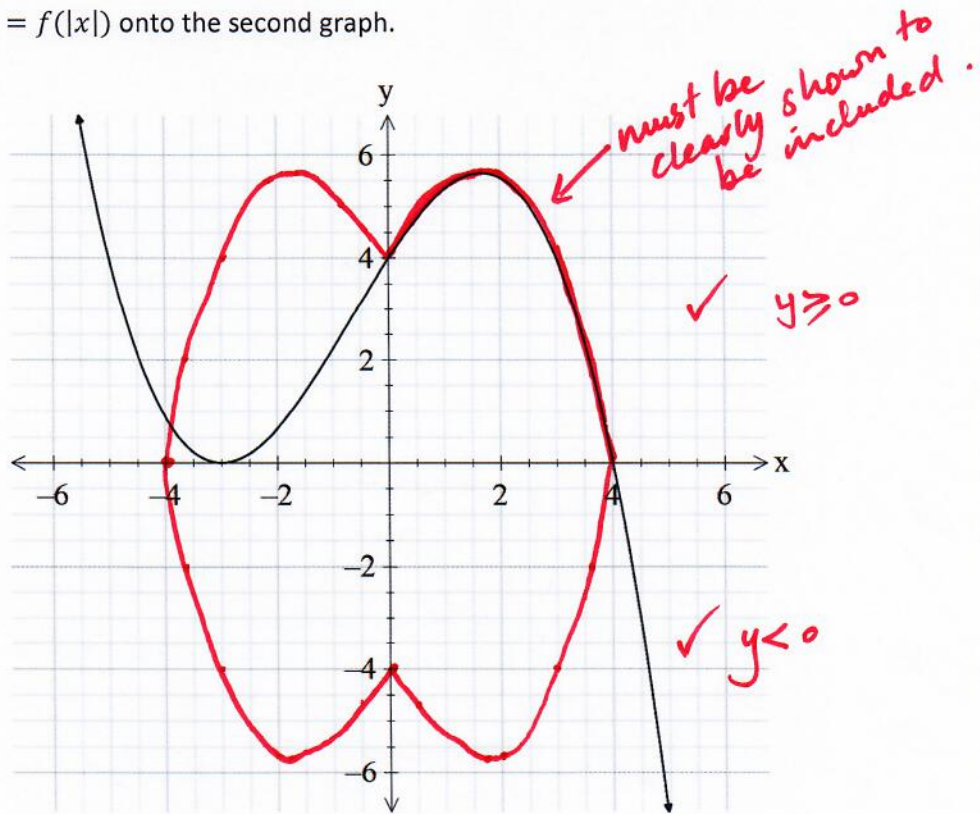
$\therefore k = 2$

horizontal asymptote.

Question 6 [2,2 = 4 marks]

The graph of $f(x)$ appears on the graphs shown below.

Add a sketch of $f(|x|)$ onto the first graph.

clearly
visible or
indicated.

Add a sketch of $|y| = f(|x|)$ onto the second graph.

must be shown to
clearly be included.

$y \geq 0$

$y < 0$

The graph of the function $f(x) = \frac{x^2-1}{x^2-3x+2}$ is discontinuous for two values of x. Identify each of these values and show what type of discontinuity each is.

$$f(x) = \frac{(x-1)(x+1)}{(x-1)(x-2)} = \frac{x+1}{x-2} \; ; \; x \neq 1$$

$$\therefore \text{Hole at } x = 1 \checkmark$$

$$\therefore x \neq 1, \; x \neq 2 \checkmark$$

$$\frac{x+1}{x-2} = 1 + \frac{3}{x-2} \qquad \text{Vertical Asymptote at } x = 2. \checkmark$$